

# 1 Algebra 1

## *What will we learn?*

In this chapter we will see how letters can be used to represent unknown numbers. Such letters are called **variables**.

We will see how such variables give us algebraic **terms**, like  $4x$  and  $2y$ .

When algebraic terms are put together by mathematical operations such as  $+$  and  $-$  we get an algebraic **expression**, like  $3x + 4y + 2$ .

We will learn how to **simplify** algebraic expressions by adding and subtracting 'like terms'.

We will also look at **factorising** expressions, which is essentially a further form of simplifying.

## *What do we already know?*

You have studied much of this for your Junior Cert, so it should be somewhat familiar.

## *Key words*

When you have completed this chapter these words should be familiar to you:

- Variable
- Term
- Expression
- Simplify
- Factorise
- Evaluate



## 1.1 Variables, Constants and Coefficients

### Variables and Constants

We use letters to represent unknown quantities. We call them **variables** because their values vary or change in different situations.

Look at the following expressions:

$$4x - 9$$

$$3a^2 - b + 5$$

The first expression contains the variable  $x$  while the second contains the variables  $a$  and  $b$ .

We say that the first expression has two **terms**,  $4x$  and  $-9$ . The second expression has three terms,  $3a^2$ ,  $-b$  and  $4$ .

The terms  $-9$  and  $5$  are **constants** because their values remain the same no matter what expression they appear in.

#### Example 1:

$$2x - 3$$

The expression has two terms. The variable in this expression is  $x$ . The constant is  $-3$ .

#### Example 2:

$$2a + b$$

The expression has two terms. The variables are  $a$  and  $b$ . There are no constants in this expression.

### Coefficients

The number accompanying a variable is called a **coefficient**.

For example, the coefficient of  $4x$  is  $4$ .

## 1.2 Simplifying Expressions

Adding and subtracting **like terms** helps us to **simplify the expression**.

For example:

$$4x + 5x = 9x$$

$$3x^2 - 2x^2 = x^2$$

$$4xy + 2xy = 6xy$$

In each case we simply add the coefficients of the like terms.

#### Example 1:

#### Solution

Simplify

$$3x^2 + 2x - x^2 + 3x$$

$3x^2$  and  $-x^2$  when added give us  $2x^2$ .

$2x$  and  $3x$  when added give us  $5x$ .

So the expression  $3x^2 + 2x - x^2 + 3x$  when simplified becomes  $2x^2 + 5x$ .

**Example 2:**

Simplify each of the following, if possible:

**Solution**

- $3x^2 + 7x^2$
  - $4xy - 2xy$
  - $3a^2 + 4a$
- $3x^2 + 7x^2 = 10x^2$
  - $4xy - 2xy = 2xy$
  - $3a^2 + 4a$  cannot be simplified since the terms are not alike.

**Example 3:**

Simplify each of the following:

- i.  $x^2 - 4xy + 3x^2 + 4y^2 + 10xy$
- ii.  $2a^2 - b^2 - 2b + a - 2b^2 + b$

**Solution i.**

	$x^2 - 4xy + 3x^2 + 4y^2 + 10xy$
Bring the like terms together	$x^2 + 3x^2 - 4xy + 10xy + 4y^2$
Adding $x^2$ and $3x^2$ gives us $4x^2$	$4x^2 - 4xy + 10xy + 4y^2$
Adding $-4xy$ to $10xy$ gives us $6xy$	$4x^2 + 6xy + 4y^2$

We now have the simplified expression.

**Solution ii.**

	$2a^2 - b^2 - 2b + a - 2b^2 + b$
Bring the like terms together	$2a^2 - b^2 - 2b^2 - 2b + b + a$
Adding $-b^2$ and $-2b^2$ gives us $-3b^2$	$2a^2 - 3b^2 - 2b + b + a$
Adding $-2b$ and $b$ gives us $-b$	$2a^2 - 3b^2 - b + a$

We now have the simplified expression.



## Exercise 1.2

State whether the following expressions can be simplified or not. Simplify where possible.

i.  $4x + 3x$

ii.  $2x + 10x + 3x$

iii.  $3a + 5a + 6a$

iv.  $3x - 5x + 2x$

v.  $4x + x^2 + x^3$

vi.  $2x^2 + 3x^2 - x^2$

vii.  $5y^2 + 3y^2 - 6y^2$

viii.  $x^3 + x^2 - 1$

ix.  $x^3 - 2x^3 + 3x^3$

x.  $5xy - 2xy - 4xy$

xi.  $4x + 10 - 3x + 2$

xii.  $2x^2 + 4x - 10x + 2$

xiii.  $x^2 - 4x + 2x + 5$

xiv.  $3x^2 + 10x - 8x - 4$

xv.  $3x^2 + 4x - 8 - x^2 + 6x + 4$

xvi.  $x^2 + y^2 + 2xy$

xvii.  $x^2 + y^2 + 2x + 2x^2 - 3y^2 - 6x$

## 1.3 Multiplication

### 1.3.1 Multiplying Terms

**Example 1:**

$(5x)(2x)$

**Solution**

Multiply 5 by 2 and  $x$  by  $x$

$(5x)(2x)$

$10x^2$

**Example 2:**

$(-5x^2)(3x^2)$

**Solution**

Multiply  $-5$  by  $3$  and  $x^2$  by  $x^2$

$(-5x^2)(3x^2)$

$-15x^4$

**Example 3:**

$(-x^2)(-7x)$

**Solution**

The coefficient of  $-x^2$  is  $-1$ .  
Multiply  $-1$  by  $-7$  and  $x^2$  by  $x$

$(-x^2)(-7x)$

$7x^3$

### Exercise 1.3.1

Multiply each of the following.

i.  $(2x)(4x)$

ii.  $(3x)(6x)$

iii.  $(x)(2x)$

iv.  $(3x)(8x)$

v.  $(12x)(9x)$

vi.  $(4x)(-2x)$

vii.  $(-5x)(6x)$

viii.  $(10x)(-8x)$

ix.  $(-3x)(13x)$

x.  $(15x)(-4x)$

xi.  $(-20x)(12x)$

xii.  $(-4x)(-2x)$

xiii.  $(-3x)(-6x)$

xiv.  $(-12x)(-5x)$

xv.  $(-25x)(-30x)$

### 1.3.2 Multiplying Terms with Brackets

If we are presented with the following,  $6(x - 4)$ , we simply multiply both terms inside the brackets by the term outside the brackets.

$$\begin{aligned} 6(x - 4) \\ = 6x - 24 \end{aligned}$$

#### Example 1:

$$2(x + y)$$

#### Solution

$$\begin{aligned} 2(x + y) \\ = 2x + 2y \end{aligned}$$

#### Example 2:

$$3(2x - 4y)$$

#### Solution

$$\begin{aligned} 3(2x - 4y) \\ = 6x - 12y \end{aligned}$$

#### Example 3:

$$5(4x - 3y) + 2(x + 7y)$$

#### Solution

Multiply out the brackets

$$20x - 15y + 2x + 14y$$

Bring the like terms together

$$20x + 2x - 15y + 14y$$

Simplify the expression by adding like terms

$$22x - y$$

### Exercise 1.3.2

For each of the following expressions, remove the brackets and simplify the answer.

i.  $2(x^2 - 3x + 2) + 3(x^2 + x - 2)$

iv.  $5(x^2 + x - 3) - 4(x^2 + 2x - 3)$

ii.  $4(x^2 + x - 10) + 2(x^2 + 2x - 3)$

v.  $3(2x^2 + 4x - 1) - 2(5x^2 + 6x + 3)$

iii.  $2(x^2 + 2x - 1) - 3(x^2 + 4x + 4)$





### 1.3.3 Multiplying Expressions

To multiply two expressions like  $(x - 4)$  and  $(x + 3)$  we multiply the contents of the second bracket by each of the terms in the first.

#### Example 1:

$$(x - 4)(x + 3)$$

#### Solution

	$(x - 4)(x + 3)$
Multiply the second bracket by the $x$ in the first	$x(x + 3)$
This gives us	$x^2 + 3x$
Multiply the second bracket by the $-4$ in the first	$-4(x + 3)$
This gives us	$-4x - 12$
Bring the two together	$x^2 + 3x - 4x - 12$
Simplify the expression by adding like terms	$x^2 - x - 12$

#### Example 2:

$$(x + 3)(x - 8)$$

#### Solution

	$(x + 3)(x - 8)$
Multiply the second bracket by the $x$ in the first	$x(x - 8)$
	$x^2 - 8x$
Multiply the second bracket by the $3$ in the first	$3(x - 8)$
	$3x - 24$
Bring the two together	$x^2 - 8x + 3x - 24$
	$x^2 - 5x - 24$

### Exercise 1.3.3

Multiply the following expressions and simplify the answers.

- |                              |                               |                                 |                           |
|------------------------------|-------------------------------|---------------------------------|---------------------------|
| <b>i.</b> $(x + 2)(x + 1)$   | <b>vi.</b> $(x + 6)(x - 4)$   | <b>xi.</b> $(2x + 3)(x + 2)$    | <b>xvi.</b> $(x + 3)^2$   |
| <b>ii.</b> $(x + 3)(x + 5)$  | <b>vii.</b> $(x + 6)(x - 5)$  | <b>xii.</b> $(3x - 4)(2x + 3)$  | <b>xvii.</b> $(x - 4)^2$  |
| <b>iii.</b> $(x + 4)(x + 4)$ | <b>viii.</b> $(x + 3)(x - 3)$ | <b>xiii.</b> $(5x - 4)(2x + 6)$ | <b>xviii.</b> $(x + 5)^2$ |
| <b>iv.</b> $(x + 3)(x - 2)$  | <b>ix.</b> $(x - 3)(x - 5)$   | <b>xiv.</b> $(3x + 2)(3x - 2)$  | <b>xix.</b> $(x - 7)^2$   |
| <b>v.</b> $(x + 4)(x - 3)$   | <b>x.</b> $(x - 6)(x - 6)$    | <b>xv.</b> $(5x - 6)(3x + 7)$   | <b>xx.</b> $(x + 10)^2$   |